

Computer modeling of multiphase fluid flow (in fractured and porous media)

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cycles. A National Laboratory workshop. LLNL December
14-16, 2005

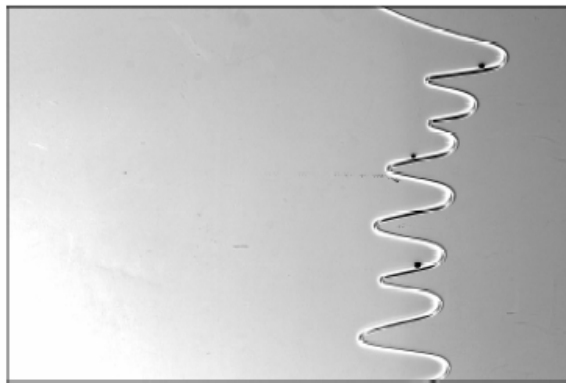
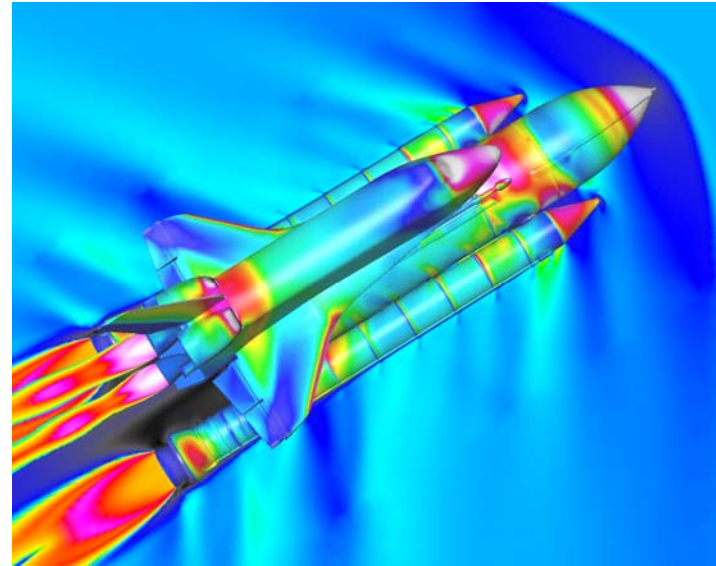
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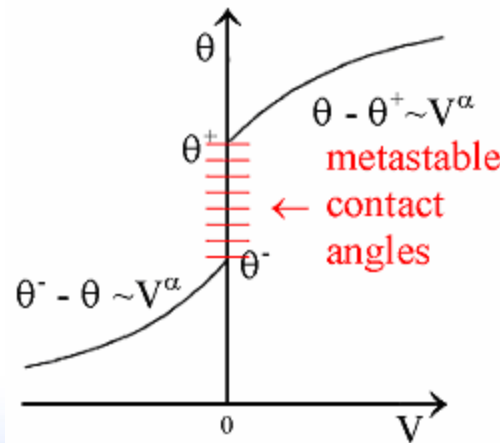
Computer simulation of multiphase fluid flow in fractured and porous media.

Two computational challenges:

1. Complex interface dynamics (topological changes)
2. Complex contact line dynamics



Veretennikov *et al.* 2005



Research Strategy

- ❖ Integrated computer modeling and experiments (but will discuss only simulations today)
 - ❖ Develop a suite of computer modeling methods with complementary strengths and weaknesses
 - ❖ Particle-based methods:
 - Lattice Boltzmann (LB)
 - Smoothed particle hydrodynamics (SPH)
 - Dissipative particle dynamics (DPD)
 - ❖ Grid-based Navier Stokes solvers with interface tracking/capturing
 - Level set interface tracking (LS)
 - Volume of fluid interface tracing (VOF)
- } Use indicator function that is advected with flow

Particle-based vs. grid-based models for multiphase fluid flow

❖ Advantages:

- Particles move with fluid (no interface tracking required)
- Rigorous mass conservation
- Reproduce complex behaviors associated with fluid-fluid-solid interface dynamics
- Relatively simple code
- Relatively easy to add additional physics

❖ Disadvantages:

- Less (sometimes much less) computationally efficient than grid-based Navier Stokes solvers.
- Fundamental fluid properties must be measured – they are controlled by particle-particle interactions



Smoothed particle hydrodynamics

- ❖ Continuous fields are represented by sum of weighting functions centered on particles:

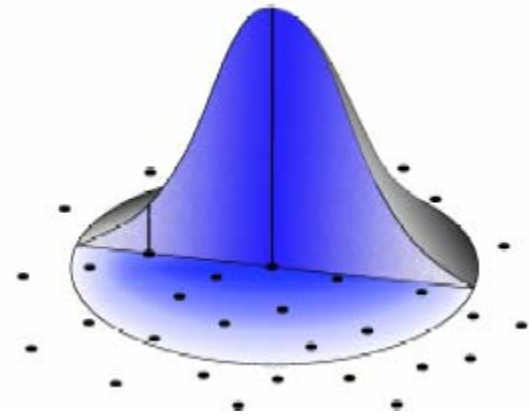
$$f(\mathbf{x}) = \sum m_i f_i W(|\mathbf{x} - \mathbf{x}_i|) / \rho_i, \quad \rho(\mathbf{x}) = \sum m_i W(|\mathbf{x} - \mathbf{x}_i|)$$

$$\nabla f(\mathbf{x}) = \sum m_i f_i \nabla W(|\mathbf{x} - \mathbf{x}_i|) / \rho_i$$

- ❖ Conservation equations:

$$d\mathbf{V}/dt = \nabla \cdot \mathbf{P} / \rho, \quad d\rho/dt = \rho \nabla \cdot \mathbf{V} \quad (\text{or calculate density from particle positions via weighting function}).$$

- ❖ Calculate pressure from density via equation of state.
- ❖ Approximate derivatives using weighting functions (compare with finite difference).
- ❖ $d\mathbf{V}/dt + = -\nabla P / \rho + \eta \nabla^2 \mathbf{V} + \mathbf{F}_{\text{ex}}$ (Navier Stokes equation)
- ❖ $d\mathbf{V}_i/dt = \sum m_j (P_j / \rho_j^2 + P_i / \rho_i^2) \cdot \nabla W(|\mathbf{x} - \mathbf{x}_j|) / \rho_i +$
- ❖ additional terms to represent effects of viscosity, interactions with solid surfaces, surface tension



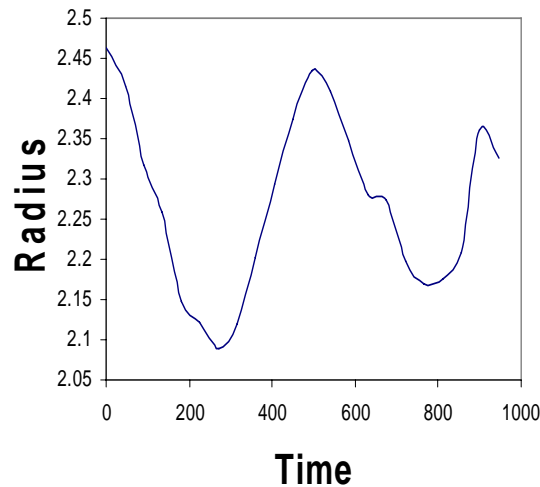
Smoothed particle hydrodynamics: Advantages and disadvantages

- ❖ Versatile - can be used to simulate fluids and solids under extreme conditions.
- ❖ Numerically unstable under some conditions (we have had no problems).
- ❖ Most applications have been in astrophysical fluid dynamics.
- ❖ Simulates continuum equations, but particle—particle interactions can be added to simulate phase separation, surface tension, wetting etc.
- ❖ Insufficient theoretical basis for these ‘hybrid models’.
- ❖ Galilean invariant (forces depend only on position and velocity differences).
- ❖ Intrinsic (particle based) contribution to fluid properties (model fluid properties are not the same as those used in the ‘governing’ equations).

Estimation of surface tension

$$\sigma = 0.18 \pm 5\%$$

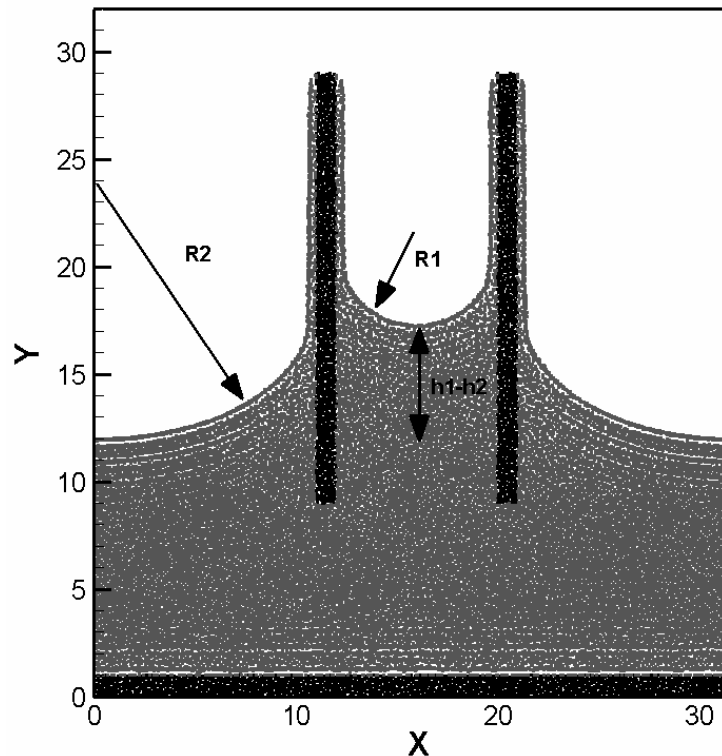
Bubble oscillation



period of
oscillation

$$\tau = 2\pi \sqrt{\frac{R^3 \rho}{6\sigma}}$$

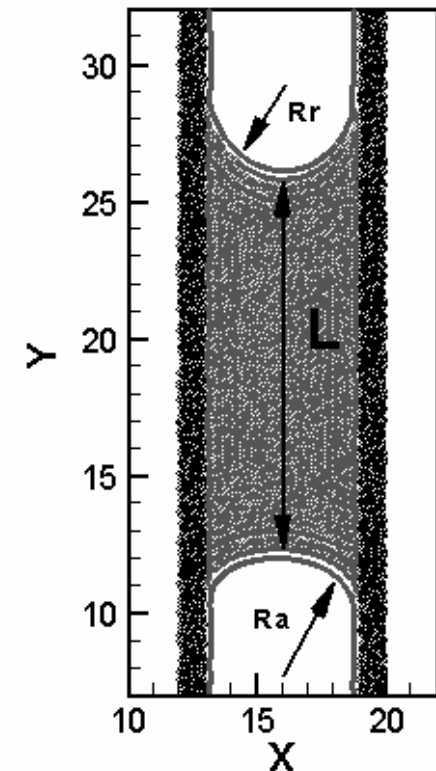
Capillary rise



ρg ↓

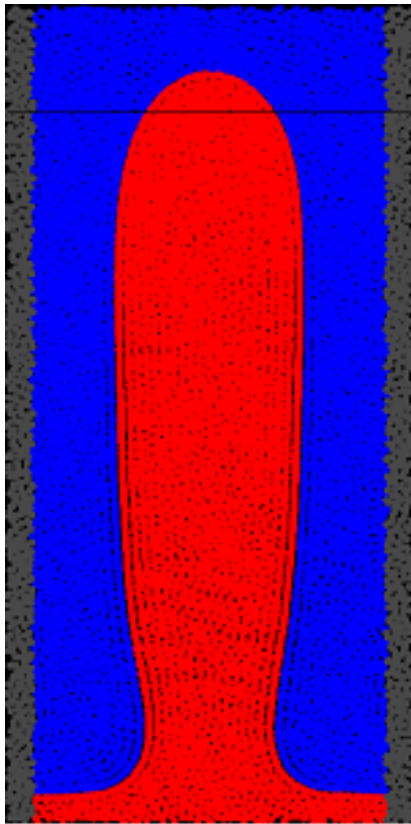
$$\rho g (h_1 - h_2) = \sigma \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Droplet in fracture

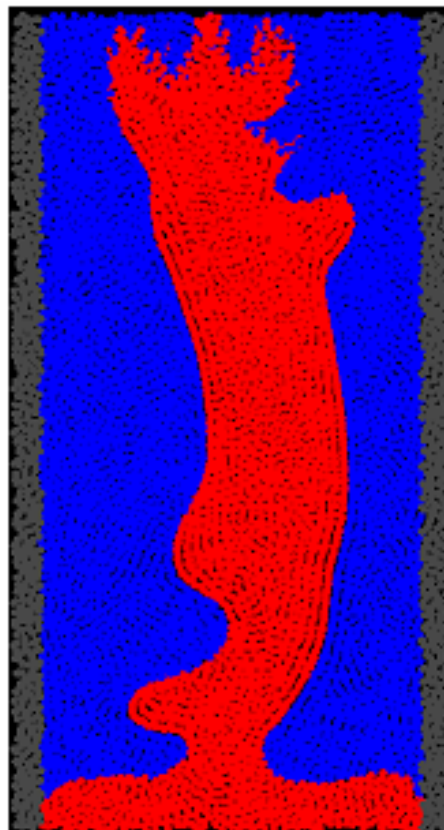


$$v = \frac{b^2}{12\mu} \left\{ \rho g + \frac{\sigma}{L} \left(\frac{1}{R_a} - \frac{1}{R_r} \right) \right\}$$

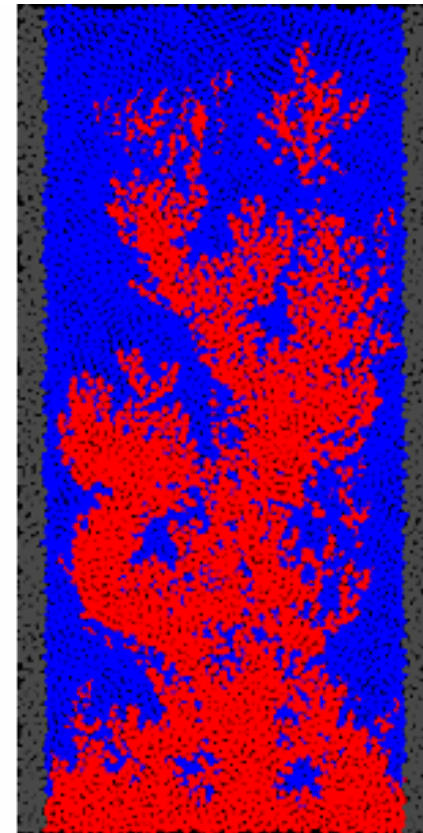
Smoothed particle hydrodynamics simulation of viscous fingers



$Ca = 0.316$



$Ca = 5.31$

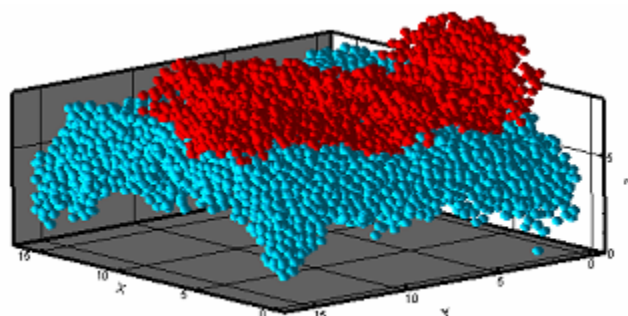
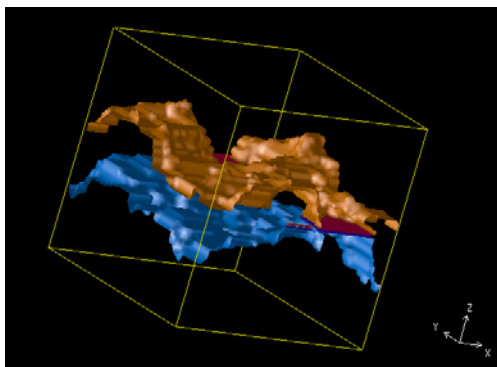


$Ca = \infty$

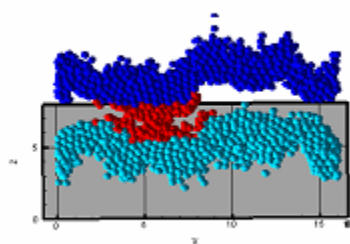
$m_k = 1, m_l = 0.2, \mu_k = 1$ and $\mu_l = 16$.

$Ca = \mu U / \Gamma$

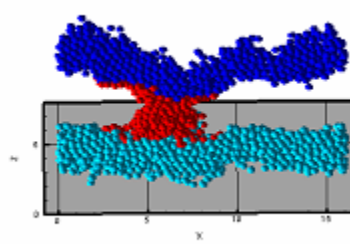
Smoothed particle hydrodynamics simulation of precipitate growth in a fracture aperture



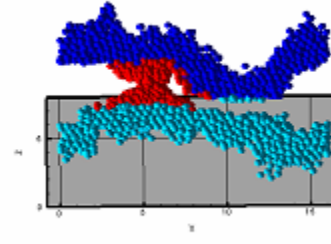
a)



b) $y = 4$



c) $y = 8$

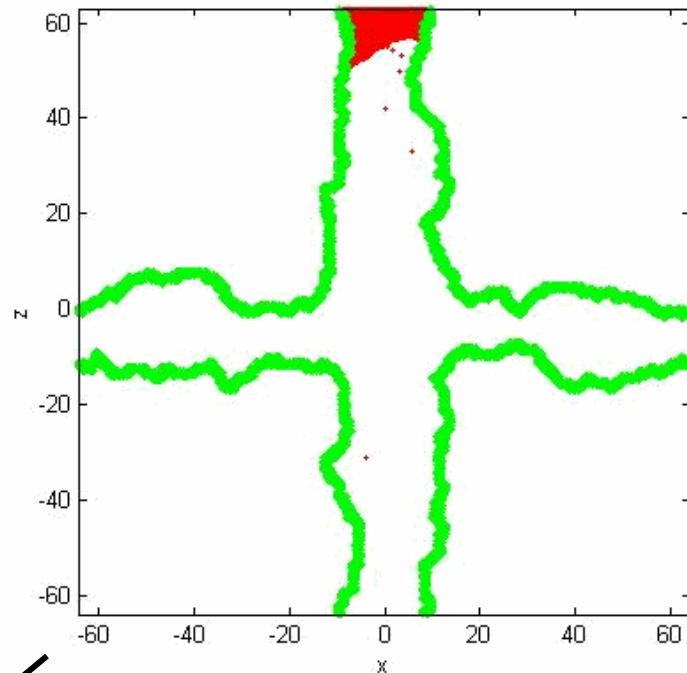


d) $y = 12$

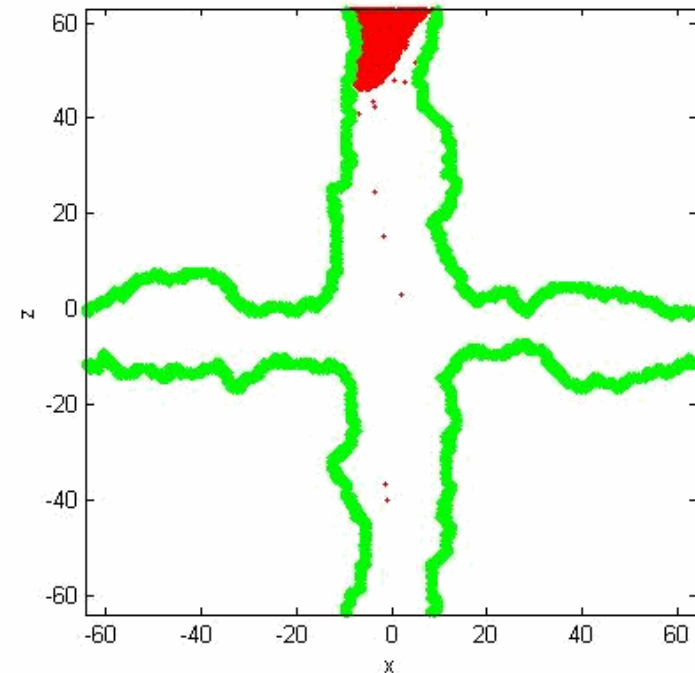
Dissipative particle dynamics: Advantages and disadvantages.

- ❖ Very versatile: can handle complex fluids and complex boundary conditions.
- ❖ Numerically stable: Dissipative and random forces act as thermostat, and deviation of kinetic temperature from fluctuation-dissipation temperature can be used to determine acceptable step size.
- ❖ Not very computationally efficient – but much better than MD.
- ❖ Soft potential \Rightarrow high compressibility.
- ❖ Includes effects of thermal fluctuations.
- ❖ Must measure model parameters – like MD.
- ❖ Galilean invariant (forces depend only on position and velocity differences)

Dissipative particle dynamics simulation of penetration of fluid into a fracture junction



Strongly wetting



Weakly wetting

Dissipative particle dynamics

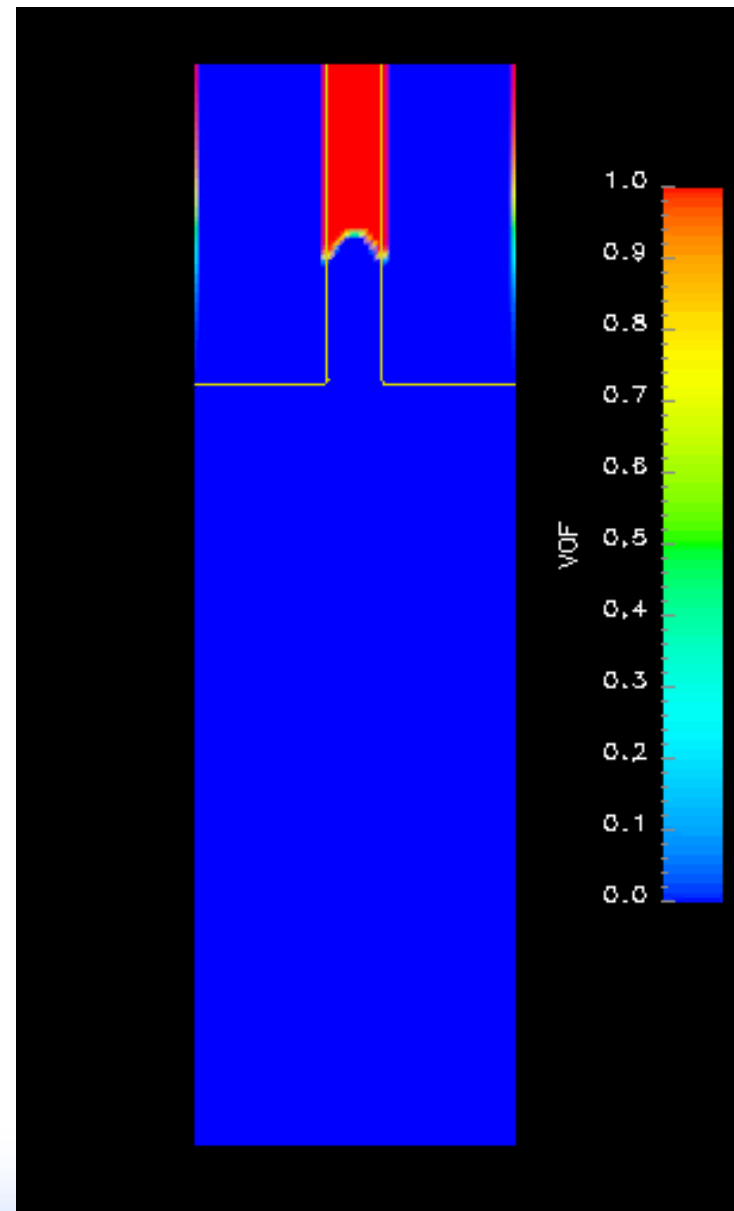
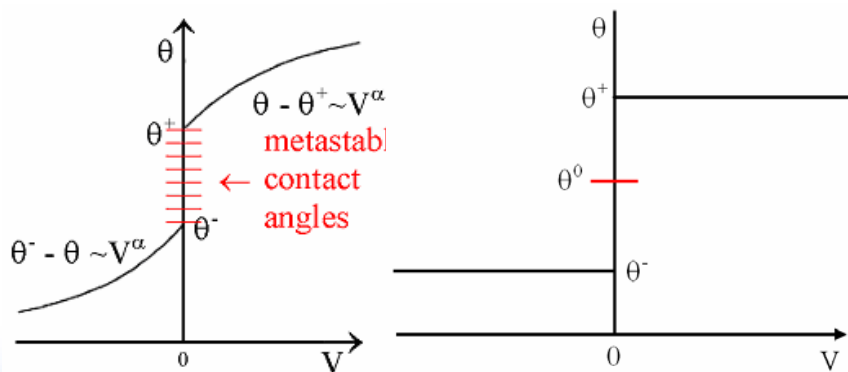
- ❖ Particles interact through conservative, dissipative and random forces
- ❖ Conservative particle-particles are soft – large time steps
- ❖ Individual particles represent (very) small fluid volumes
- ❖ Interactions rigorously conserve momentum
- ❖ Fluctuation-dissipation relationship between random and dissipative interactions
- ❖ Algorithm is like thermostatted molecular dynamics
- ❖ Short range repulsive + long(er) range attractive interactions lead to phase separation

Navier Stokes solvers with indicator function interface capturing

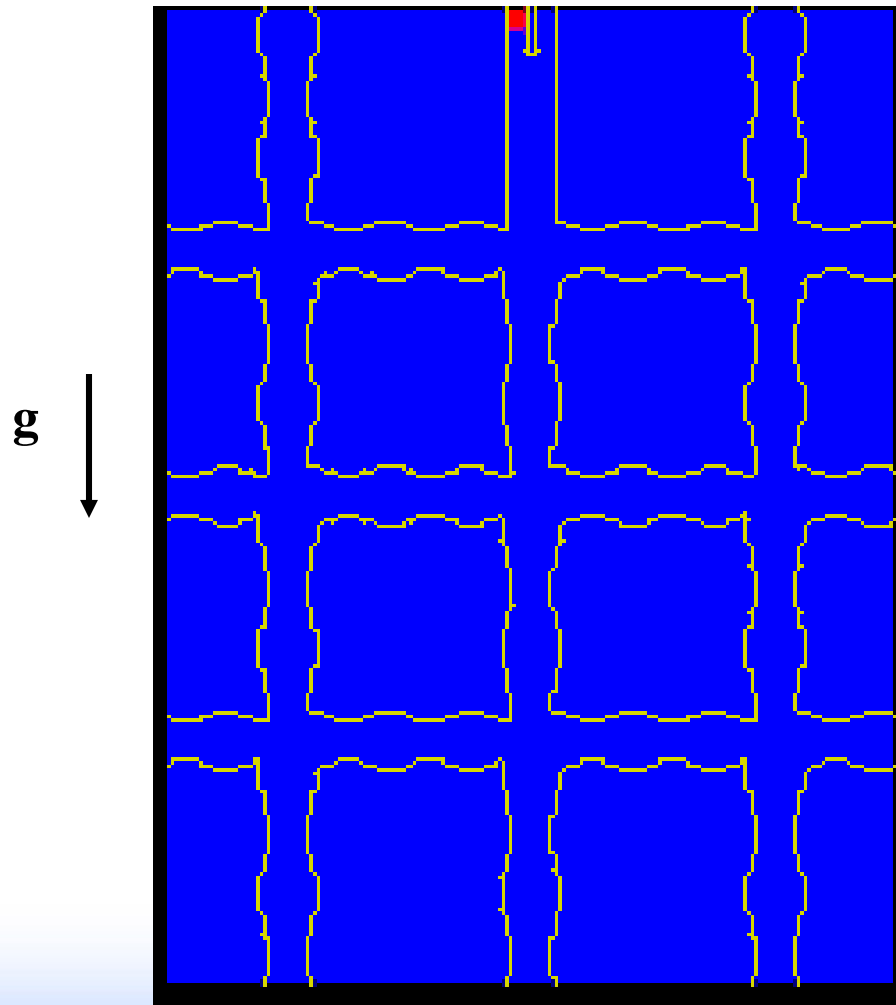
- ❖ Numerical solution of Navier Stokes equation.
- ❖ Conservation of material: $\nabla \bullet \mathbf{V} = 0$
- ❖ Conservation of momentum:
$$\rho d\mathbf{V}/dt = -\rho (\mathbf{V} \bullet \nabla) \mathbf{V} - \nabla P + \eta \nabla^2 \mathbf{V} + \mathbf{F}_{ex}$$
- ❖ Level set interface capturing: Fluid-fluid interface is the 'level-set' (zero level cut) of the level set function, $\phi(\mathbf{x})$, which is advected with the flow.
- ❖ Volume of fluid interface tracking: The indicator function, $\phi(\mathbf{x})$, which is advected with the flow, has a value of 1 in fluid 1, a value of 0 in fluid 2, and $0 < F < 1$ for grid cells that contain part of the fluid-fluid interface.

Simulation of dripping
faucets: Complex chaotic
dynamics (deterministic
chaos)

Three-dimensional (two-
dimensional with axial
full-rotation symmetry)

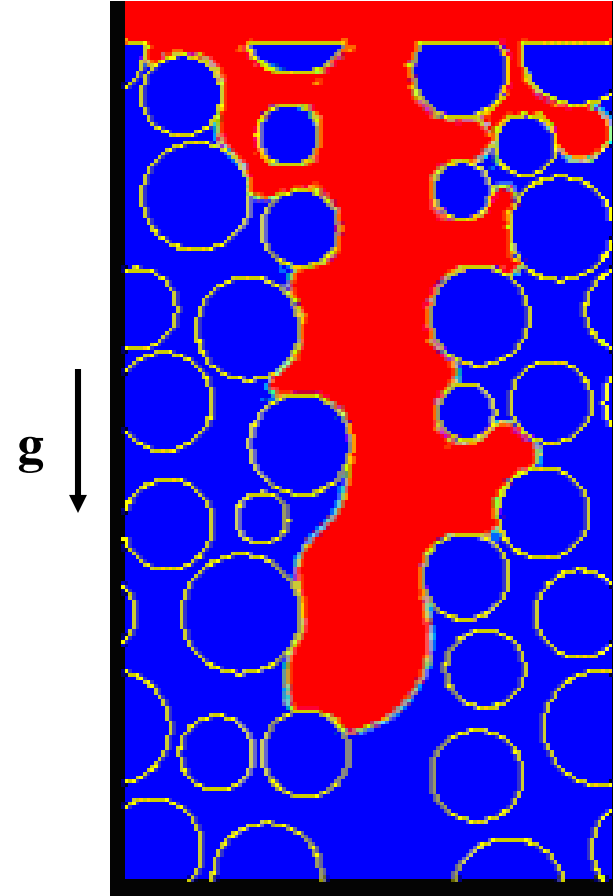
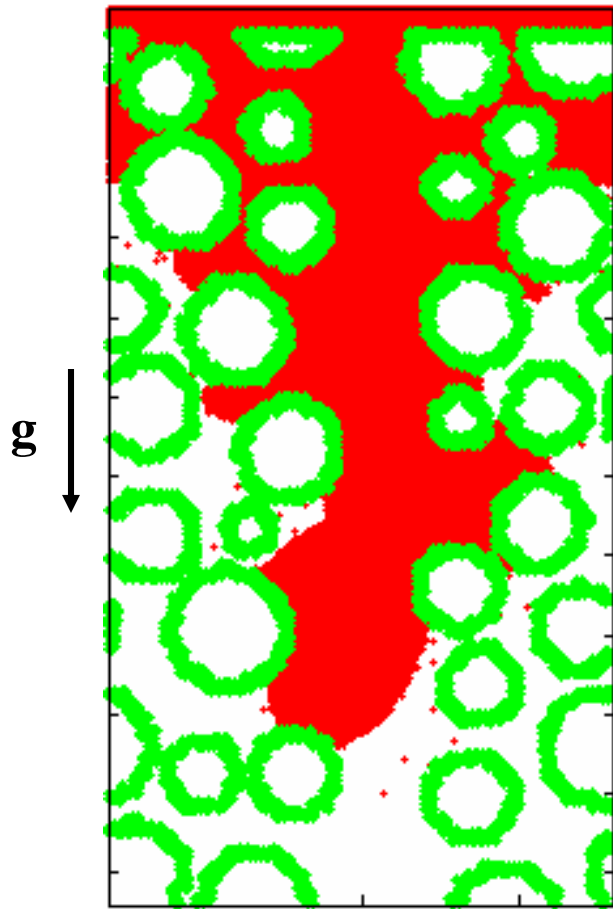


Simulation of multiphase fluid flow in fracture junction with VOF interface capturing



H. Huang, P.Meakin, and M. Liu,
Geophysical Research Letters, **32**:
L19402, doi:10.1029/2005GL023899.
(2005)

Dissipative particle dynamics vs. grid-based simulation – penetration into fractured porous medium



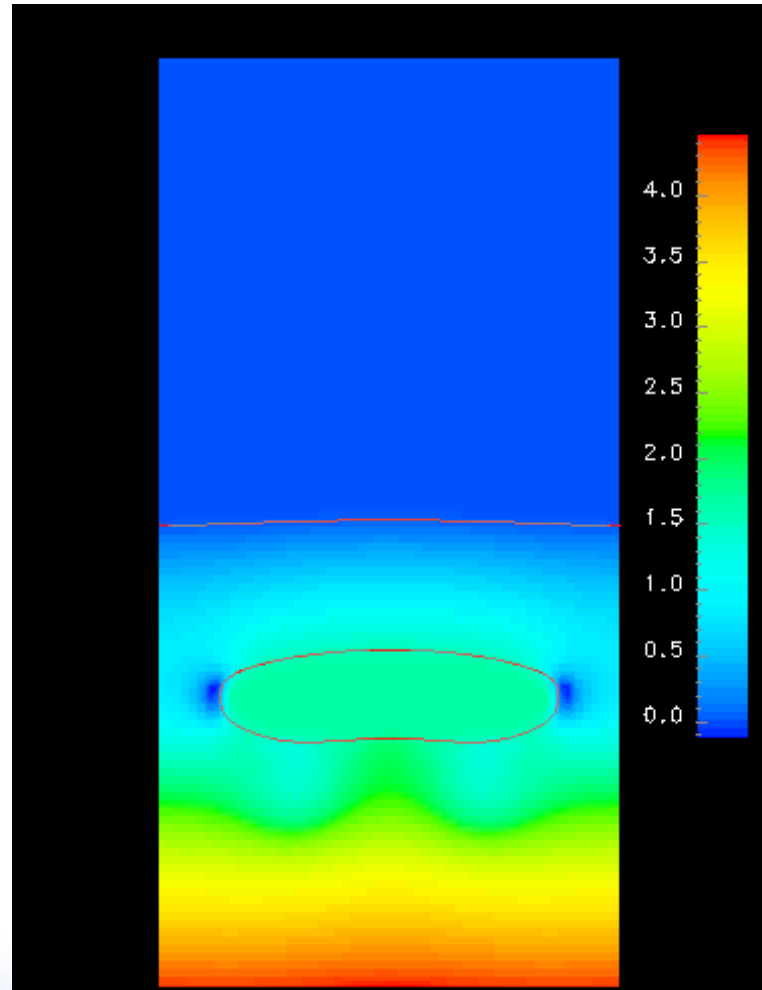
Finite volume -VOF

Level set interface capturing in combination with finite difference methods

- High computational efficiency
- Can handle large density and viscosity contrasts (water/air)
- Some mass loss: controllable
- Difficult to handle solid/water/air contact line dynamics: current challenge
- Most simple and elegant grid-based method for interface dynamics

Air bubble bursting near water surface – Level set interface tracking

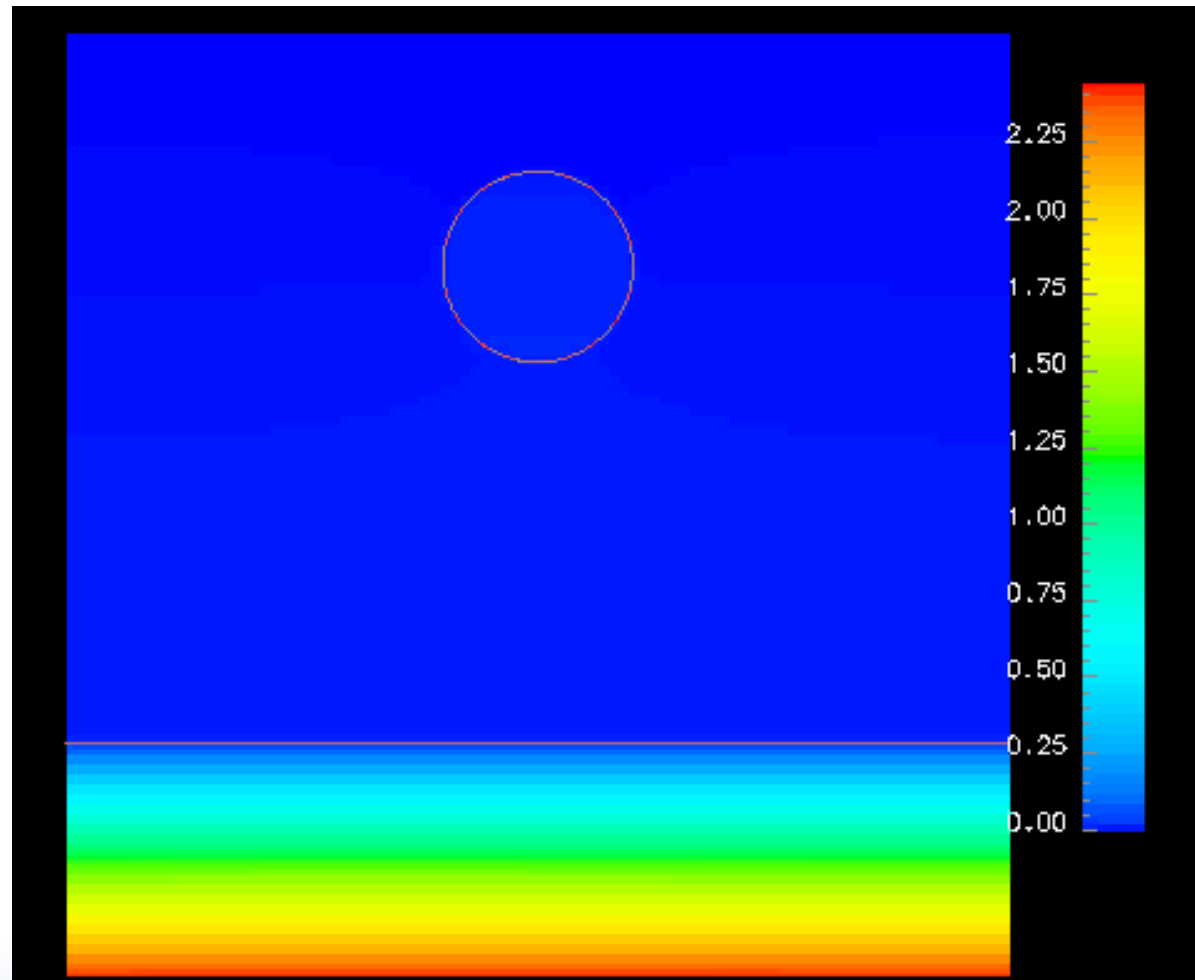
Three-dimensional
(two-dimensional
with axial full-
rotation
symmetry)



Falling water droplet impacting on water surface – Level set interface tracking

Three-
dimensional
(two-
dimensional
with axial
full-rotation
symmetry)

g ↓



Going forward

- ❖ High performance computing (2-d \rightarrow 3-d)
- ❖ Hybrid particle/continuum multiscale modeling (use particle based model near interfaces and more computationally efficient grid-based continuum model in bulk)
- ❖ Multiscale simulations (particles of different sizes ...)
- ❖ Better theory for particle-based models:
 - (relationship between particle-particle interactions, wetting behavior & fluid properties)
 - Spurious velocity fluctuations near interfaces
 - Theoretical foundation for multiscale modeling

